ENEE 661 SPRING 2017 Homework Set 1
(due back in class Tuesday February 7).

1. We say that a group G acts (on the left) on a set S if there is map

$$\Phi : G \times S \to S$$

$$(g, p) \mapsto \Phi(g, p)$$

with the properties

$$\Phi(g_1, \Phi(g_2, p)) = \Phi(g_1 g_2, p)$$

$$\Phi(e, p) = p$$

where e denotes identity element of group. We say the action is <u>transitive</u> if given $p, q \in S$ there is a $g \in G$ such that

$$\Phi(g, p) = q$$

Show that the group $SE(n)$ acts transitively, on the left on $\mathbb{R}^n$ by the map

$$\Phi\left(\begin{pmatrix} A & b \\ 0 & 1 \end{pmatrix}, x\right) = Ax + b$$

2. Let $Sl(n)$ denote the group of $n \times n$ nonsingular matrices of determinant 1. Let $t \mapsto g(t)$ denote a differentiable curve in $Sl(n)$. Show that we can write

$$\dot{g}(t) = g(t)\,\xi(t)$$

where $\xi(t)$ is trace-free.

3. For a curve $t \mapsto \gamma(t) \in \mathbb{R}^3$ show that curvature and torsion can be written as

$$\kappa = \|\dot{\gamma} \times \ddot{\gamma}\| \Big/ \|\dot{\gamma}\|^3$$

$$\tau = \dot{\gamma} \cdot (\ddot{\gamma} \times \dddot{\gamma}) \Big/ \|\dot{\gamma} \times \ddot{\gamma}\|^2$$

Hint: You need to convert from arc-length parametrization (as in Lecture 2) back to time parametrization of γ

4. Let us denote as $Rat(n)$ the set of all rational transfer functions (of linear systems) of the form

$$T = T(s) = \frac{q(s)}{p(s)} = \frac{q_{n-1} s^{n-1} + \cdots + q_0}{s^n + p_{n-1} s^{n-1} + \cdots + p_0}$$

Consider transformations on the set of all transfer functions of degree n defined by

$$\Phi\left((\alpha, k), T \right) = \frac{\alpha T}{1 + \alpha k T}$$

interpreted as magnitude scaling $(\alpha \neq 0)$ and feedback $(k \in \mathbb{R})$.

Show that there is a matrix group G with elements g uniquely associated to α and k for which Φ defines a group action. Show further that this is a non-commutative group.