ENEE 664 Spring 2004 Homework Set 2.

| Due back in class |
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| Thursday 02.12.04 |

1. Consider the problem of minimizing the quantity

$$\eta = \int_{t_0}^{t_1} (u'(t) \;\; x'(t)) \begin{bmatrix} 1 & N(t) \\ N'(t) & L(t) \end{bmatrix} \begin{pmatrix} u(t) \\ x(t) \end{pmatrix} dt \quad + \; x'(t_1) Q x(t_1)$$

for the system $\dot{x}(t) = A(t)x(t) + B(t)u(t)$. Show that (in the spirit of the fundamental lemma on path independent integrals), the appropriate Riccati equation is

$$\dot{K}(t) = -A'(t)K(t) - K(t)A(t) - L(t) + (N(t) + B'(t)K(t))'(N(t) + B'(t)K(t))$$

and prove an analog of Theorem 1 (~~fixed~~ free end-point problem) proved in class.

2. Given the homogeneous linear differential equation

$(*) \quad \dot{x}(t) = A(t)x(t),$

the adjoint equation associated to this equation is

$(**) \quad \dot{p}(t) = -A'(t)p(t).$

Verify (i) $x'(t)p(t) = x'(t_0)p(t_0) =$ a constant;

(ii) The transition matrix of $(*)$ satisfies

$$\frac{d}{dt}\Phi_A(t_0, t) = -\Phi_A(t_0, t)A(t).$$

(iii) what is the transition matrix $\Phi_{-A'}(t, t_0)$ for $(**)$ in terms of that for $(*)$?

Show that there is ~~exist~~ an initial state $p(t_0)$ for $(**)$ such that the control $u_0$ which minimizes

$$\eta = \int_{t_0}^{t_1} u'(t)u(t)\,dt \quad + \; x'(t_1)Qx(t_1)$$

along trajectories of $\dot{x}(t) = A(t)x(t) + B(t)u(t)$ is given by $u_0(t) = -B'(t)p(t)$.

3. Consider the system $\dot{x}(t) = A(t)x(t) + B(t)u(t)$.

Suppose we want to keep $x(t)$ close to the trajectory $d(t)$, where $d(t)$ is a solution to the homogeneous equations $\dot{x}(t) = A(t)x(t)$. To do this, we define the cost functional

$$J = \int_{t_0}^{t_1} \left[u'(t)\,u(t) + (x(t)-d(t))'\,L(t)\,(x(t)-d(t))\right] dt$$

Show that the control $u_0$ which minimizes $J$ has the form

$$u_0(t) = F(t)\,x(t) + g(t).$$

What are $F(t)$ and $g(t)$?

4. Find $u$ such that the scalar system

$$\dot{x}(t) = -x(t) + u(t)$$

is driven from $\underbrace{x=1 \text{ at } t=0}$ to $\underbrace{x=0 \text{ at } t=1}$ and

$$\eta = \int_0^{1/2} u^2(t)\,dt + 2\int_{1/2}^{1} u^2(t)\,dt$$

is minimized.