ENEE 664 Spring 2004 Homework Set 3. due back in class 02.19.04

1. What is the minimum value of $\int_0^{\pi} [\dot{x}(t)]^2 \, dt$ for x satisfying $x(0) = 1$ and $x(\pi) = 0$?

2. For the time invariant system

$$\dot{x} = A x + B u + C v$$

with a known disturbance $V(t)$, suggest/derive a control that drives the system from $(x_0, 0)$ to $(0, T)$ and minimizes

$$\eta = \int_0^T u'(\sigma)\, u(\sigma) \, d\sigma$$

State any needed hypotheses clearly.

3. Complete the proof the necessary conditions theorem in Lecture Notes 3 (page 5, part (c)).

4. Consider the problem of finding an optimal control for the system: $\dot{x} = -\alpha x + u \qquad \alpha > 0;$

cost $\eta = \int_0^1 (x^2 + u^2)\, dt + x_1^2(1)$; with initial $x(0) = 1$.

Use a numerical approach based on canonical equations (and MATLAB) to investigate this problem.
What is the optimal value η_{min} ?