Homework Set # 6

Due in my office

Friday, March 20, 2004.

1. In the following assume $x(t)$ to be continuously differentiable. Use arguments based on control theory or calculus of variations to show that

(a) $x(0) = 0$

$$\Rightarrow \int_0^{\pi/2} [\dot{x}(t)]^2 dt \geq \int_0^{\pi/2} [x(t)]^2 dt$$

(b) $x(0) = 0 = x(\pi)$

$$\Rightarrow \int_0^{\pi} [\dot{x}(t)]^2 dt \geq \int_0^{\pi} [x(t)]^2 dt$$

(c) $x(0) = \dot{x}(0) = x(\pi) = \dot{x}(\pi) = 0$

$$\Rightarrow \int_0^{\pi} [\ddot{x}(t)]^2 dt \geq \int_0^{\pi} [x(t)]^2 dt$$

(d) $\int_0^{2\pi} x(t)\,dt = 0$ and $x(0) = 0 = x(2\pi)$

$$\Rightarrow \int_0^{2\pi} [\dot{x}(t)]^2 dt \geq \int_0^{2\pi} [x(t)]^2 dt$$

2. For a problem of calculus of variations with $J[x] = \int_{t_1}^{t_2} L(t, x, \dot{x})\, dt$ with t_1 and t_2 given and $x(t_1) = x_1$ given, but $x(t_2)$ unspecified, determine the first order necessary conditions for extrema.

3. Read the Sussmann-Willems paper on the Brachistochrone upto and including the end of the section on Hamilton (page 40). (additional assignment to come!)