ENEE 664 Homework Set 2. Spring 2012
[due back: February 9]

1. Let $W(t_0, t_1)$ denote the Gramian (see page 7 Lecture Notes 1) for the accessibility question.

If it is invertible for $t \in [t_0, t_1)$ show that the inverse $K(t) = W(t, t_1)^{-1}$ satisfies the Riccati differential equation

$$\frac{d}{dt} K = - A^T(t)\, K(t) - K(t)\, A(t) + K(t)\, B(t) B^T(t)\, K(t)$$

show also that

$$W(t_0, t_1) = W(t_0, t) + \Phi(t_0, t)\, W(t, t_1)\, \Phi^T(t_0, t)$$

2. In the capacitor charging problem of the example on page 9 of the notes, how does the efficiency change when a series inductor L is introduced?

inductance

3. Consider the modification to the cost η of the free end point problem of Lecture Notes 2 given as,

$$\eta = \int_{t_0}^{t_1} [u'(t) \;\; x'(t)] \begin{bmatrix} 1 & N(t) \\ N'(t) & L(t) \end{bmatrix} \begin{bmatrix} u(t) \\ x(t) \end{bmatrix} dt$$

$$+ \quad x^T(t_1)\, Q \, x(t_1)$$

for the system $\dot{x}(t) = A(t)x(t) + B(t)u(t)$, $x(t_0) = x_0$.

Mimicking Theorem 1 of Lecture Notes 2 (see page 2) derive a formula for optimal control.
[will discuss relevant material in class on February 7]

4. Find $u(\cdot)$ such that the scalar system

$$\dot{x} = -x + u$$

is driven from $x_0 = 1$ at $t_0 = 0$ to $x_1 = 0$ at $t_1 = 1$ and the cost

$$\eta = \int_0^{1/2} u^2(\sigma)\, d\sigma + 2\int_{1/2}^{1} u^2(\sigma)\, d\sigma$$

is <u>minimized</u>