ENEE 664 Spring 2012 Homework #5 due Thursday March 1 2012

1. Consider $f : \mathbb{R}^2 \to \mathbb{R}^1$ defined by

$$f(x_1, x_2) = \begin{cases} \dfrac{x_1 x_2^2}{x_1^2 + x_2^4} & x_1 \neq 0 \\ 0 & x_1 = 0 \end{cases}.$$

Show that f is Gateaux differentiable but <u>not</u> continuous at $x_1 = x_2 = 0$.

2. Consider $f : \mathbb{R}^2 \to \mathbb{R}^1$ defined by

$$f(x_1, x_2) = \begin{cases} 0 & \text{if } x_1 = 0 \\ \dfrac{2x_2 e^{-\frac{1}{x_1^2}}}{x_2^2 + \left(e^{-\frac{1}{x_1^2}}\right)^2} & \text{if } x_1 \neq 0. \end{cases}$$

Show that f is Gateaux differentiable but <u>not</u> Fréchet differentiable at $(0,0)$.

3. Define $f : \mathbb{R}^2 \to \mathbb{R}^1$ by

$$f(x, y) = \operatorname{sgn}(y) \min(|x|, |y|).$$

For any $h \in \mathbb{R}^2$, show that

$$\lim_{t \to 0} \frac{1}{t}\left(f(th) - f(0)\right) = f(h).$$

Is f Fréchet differentiable?

4. Let $X = C[0,1]$ with $\|x\| = \max_{0 \le t \le 1} |x(t)|$

Let $f : X \to \mathbb{R}$ be defined as

$$f(x) = \left(x\left(\tfrac{1}{2}\right)\right)^2$$

Find the Fréchet differential of f.

5. Consider the mapping defined on symmetric matrices by

$$K \longmapsto f(K) = A^T K + KA - KBB^T K + L$$

where A, B, $L = L^T$ are given. Compute the Fréchet derivative of f at K_0, denoted by $Df(K_0)$.

When is it invertible?