ENEE 664 Spring 2014
due in class 02/12/2014

1. Let $W(t_0, t_1)$ be as in Lecture Notes 1. Show that

$$W(t_0,t_1) = W(t_0,t) + \Phi(t_0,t)\, W(t,t_1)\, \Phi'(t_0,t)$$

where ($'$) denotes matrix transpose, with $t_0 \leq t < t_1$.

2. Consider the modification to the cost η of the free end-point problem of lecture Notes 2 given by,

$$\eta = \int_{t_0}^{t_1} \left(u'(t) \;\; x'(t)\right) \begin{pmatrix} \mathbb{1} & N(t) \\ N'(t) & L(t) \end{pmatrix} \begin{pmatrix} u(t) \\ x(t) \end{pmatrix} dt + x'(t_1)\, Q\, x(t_1)$$

for the system $\dot{x}(t) = A(t)x(t) + B(t)u(t)$, $x(t_0) = x_0$. Derive a formula for optimal control (Hint: Mimic Theorem 1 of Lecture Notes 2)

3. Find $u(\cdot)$ such that the scalar system

$$\dot{x} = -x + u$$

is driven from $x_0 = 1$ at $t_0 = 0$ to $x_1 = 0$ at $t_1 = 1$ and the cost

$$\eta = \int_0^{1/2} u^2(\sigma)\,d\sigma + 2\int_{1/2}^{1} u^2(\sigma)\,d\sigma$$

is minimized.

4. What is the minimum value of

$\int_0^{\pi} [\dot{x}(t)]^2\,dt$ for $x(\cdot)$ satisfying

$x(0) = 1$ and $x(\pi) = 0$?