Homework Set 8 ENEE664 Spring 2014
due back in class May 7, 2014.

1 Show that the linear map $x \mapsto Ax$ defined by the matrix $A = [a_{ij}]$ with

$$|a_{ii}| > \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}| \qquad i = 1, 2, \cdots, n$$

(the diagonal dominance property), is a contraction. What is the corresponding contraction coefficient $\rho < 1$?

2. For the problem of finding $\sqrt{b}$ for a range of values of $b > 0$, carry out a comparison of the Banach iteration algorithm (Lecture notes 8, page 5) with the Newton algorithm Lecture 9(a). [Write MATLAB code for both algorithms, compare over meaningful ranges of b, plot results, submit code and plots with your solutions].

Discuss behavior when $0 < b < 0.1$ by considering several values in this range and convergence to $\sqrt{b}$.

3. For a nonlinear map $T : X \to Y$ of normed linear spaces, the second Fréchet derivative at $x \in X$

$$D^2T(x) = (D^2T)_x : X \times X \to Y$$
$$(h, k) \longmapsto D^2T(x)(h,k)$$
$$= \frac{d^2 T(x + th + sk)}{ds\,dt}\Bigg|_{\substack{s=0\\t=0}}$$

The <u>second variation</u> of T at x with increment h is
$$D^2T(x)(h,h)$$

For the functional $J[x] = \int_{t_1}^{t_2} L(t, x(t), \dot{x}(t))\,dt$

<u>Compute</u> the second variation $D^2J[x](h,h)$ when $x(t)$ is differentiable, with fixed end points $x(t_1) = x_1$ and $x(t_2) = x_2$ and show that it takes the form

$$D^2J[x](h,h) = \int_{t_1}^{t_2} \left(P(t)\,h^2(t) + Q(t)\,\dot{h}^2(t)\right)dt$$

Write down the functions $P(t)$ and $Q(t)$.

4. Let X be a Banach space. Let $A : X \to X$ be a bounded linear operator. Suppose $\| A \| = a < 1$. Show the $(\mathbb{1} - A)$ is a bounded invertible map with bounded inverse satisfying

$$\| (\mathbb{1} - A)^{-1} \| < \frac{1}{1-a} .$$

5. In problem 2 above, replace the Newton algorithm by the modified Newton algorithm

$$x_{n+1} = x_n - \lambda_n (DP(x_n))^{-1} P(x_n)$$

where λ_n is a step-size parameter selected according to the Armijo step size rule [read section 4.8 especially page 75] for the Armijo-Newton algorithm AFTER reading section 4.2 and section 4.3, especially page 55 and 56, in the lecture notes (posted) by Dr. André TITS.

Carry out the numerical comparison with the results of problem 2 above using the modified Newton method. Which algorithm is faster?