ENEE 664: Optimal Control (3 credits) spring 2016 MW 5:00pm – 6:15pm CSI 1122

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In this course, we cover the basics of the theory of optimization of continuous time, finite dimensional, dynamical systems with control inputs. We begin with the problem of linear systems with quadratic cost functionals, solving it by use of the matrix Riccati system. We then sketch the elements of *very basic calculus* in Banach spaces to state and prove the basic theorems of equality-constrained optimization. We follow this with a discussion of optimization in the presence of inequality constraints. This is followed by an introduction to the calculus of variations and its relation to problems of optimal control. In support of numerical computation of optimal solutions, we develop the basic algorithms of the subject – gradient and Newton methods. We then derive the second order necessary conditions of the calculus of variations as a point of departure for the intuitive development of Pontryagin's maximum principle (PMP) that governs the necessary conditions for an optimal control. Finally we give an outline of the theory of sufficient conditions and the Hamilton-Jacobi-Bellman equations (dynamic programming). Through various stages in the course optimal controls will be discussed in open loop and feedback forms, and the relationships between these forms will be highlighted. Connections to Lagrangian and Hamiltonian mechanics will be brought out. (**See detailed breakdown of the material on a roughly weekly basis given in the next page.**)

Of special interest: This course may be of interest to people engaged in the study of optimality principles for inverse problems in physics, engineering and biology, sensory-motor interactions (in NACS or Kinesiology), and in other areas of biology concerned with power constraints, efficiency, and evolutionary optimality.

Optional Topics: Explicitly solvable problems of optimal control; dynamic programming; solution to certain combinatorial optimization problems by differential equations of gradient type in spaces of matrices; singular optimal control and higher order necessary conditions.

Course Prerequisite: Math 410 and ENEE 660 (can be waived by instructor consent).
Topic Prerequisite: Advanced calculus (Math 410 or equivalent; Math 411 preferred); linear system theory or linear forced ordinary differential equations (mostly time-domain aspects).
Some pre-requisite material may be picked up through self-study (contact instructor for recommendations).

References: For current version of Lecture Notes (and additional references) see: http://www.ece.umd.edu/class/enee664.S2016/ The Notes will be updated for spring 2016.

Grading: Weekly homework sets will be collected and graded. Submit individual work but discussions of homework problems are encouraged. There will be two mid-term examinations and a final examination: (i) first mid-term on Wednesday, **March 9** (in class, closed book); (ii) second mid-term (in class, closed book) on Wednesday, **April 13**; (iii) final examination (in class **Monday, May 16, closed book, 4:00 – 6:00 pm; and open book 7:00 – 9:00 pm**, – proctored in assigned room). The breakdown in weighting towards the final grade will be: homework 10%, mid-terms 25% each, and finals 40%.

ENEE 664 Optimal Control (Spring 2016) – See the current version of lecture notes available at website for the course. The breakdown given below on approximately weekly basis encapsulates the notes. (Items in red refer to topics which will be covered in the stated priority order, with adjustments based on time available.)

1. Linear systems and quadratic optimal control – basics of linear systems, controllability, gramians, **optimal transfer of state**; the problem of time-optimal control and the bang-bang principle (priority 1(b))
2. Fixed end-point and free end-point problems of linear-quadratic optimal control; path independence lemma and Riccati differential equations; dynamic programming and principle of optimality.
3. Adjoint equations; linear Hamiltonian systems and Riccati equations; application to free boundary linear-quadratic optimal control problems and data fitting (priority 3)
4. Application: Derivation of the Kalman-Bucy filter as the solution to a dual optimal control problem (priority 2)
5. Function spaces and functionals; calculus on function spaces; norms, induced norms, convergence, completeness; Gateaux and Fre'chet derivatives; constrained extrema of a functional on linear space – the case of **equality constraints**; Lagrange multiplier theorem.
6. Curves in finite dimensional linear spaces, path functionals, and the **calculus of variations**; necessary conditions for fixed end-point problems and the Euler-Lagrange equations; Legendre transform and Hamilton's equations; conservation laws; Noether's theorem.
7. Free end-points and transversality; use of the Lagrange multiplier theorem in the calculus of variations; **applications -** Wirtinger type inequalities; isoperimetric inequality; **Dido's problem**; image analysis (priority 5)
8. Iterative algorithms and contraction mapping fixed point theorem; optimization problems with inequality constraints (priority 1(a))
9. Newton's method for solving nonlinear equations; main convergence theorem; applications to solving optimization problems.
10. Newton's method, convergence rate and comparison with gradient descent method; stopping rules for gradient methods; conjugate gradient method.
11. Second order necessary conditions for minimization; Taylor's formula with remainder; second order sufficient conditions; second order necessary conditions in the calculus of variations; **strengthened Legendre condition; conjugate points and sufficiency**; another route to Riccati equations.
12. **Pontryagin's** maximum principle, an introduction; Hamilton's equations; **applications** to time-optimal control, brachistochrone problem; extensions to game theory and the equation of Isaacs (priority 4)
13. Sufficiency in optimal control and the Hamilton-Jacobi-Bellman equation; verification theorem; specialization to the linear-quadratic setting.
14. **A** brief discussion of vistas (priority 6)

Policy on Classroom Environment

It is of utmost importance to maintain a classroom environment conducive to focus on and attention to instruction. Hence usage of electronic devices (music equipment, cell phones, text messaging devices and computers) is disallowed during regular class hours. Students interested in taking notes using tablet computers or in audio recording should seek permission of instructor.