Homework Set 3 due back in class February 22, 2016

Problem 1

Consider the scalar system

$$\dot{x} = u.$$

In the interest of keeping the state regulated about $x = 0$, it is natural to consider the problem of *minimizing* the cost

$$\int_0^T \left(u^2(t) + Lx^2(t)\right)\, dt$$

where L is a constant.

Suppose $L = -1$. Can you find a feedback law to solve the minimization problem? Write it down explicitly, along with any needed hypotheses. Is your answer well-behaved as the time horizon T is increased, from $T = 1$ to $T = 2$?If so, determine optimal costs for both time horizons.

Problem 2

Consider the linear time-varying system

$$\dot{x}(t) = A(t)\,x(t) + B(t)\,u(t)$$

where

$$A(t) = \begin{pmatrix} a(t) & 0 \\ 0 & a(t) \end{pmatrix}; \quad B(t) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

Is there a control that drives the system from

$$x(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

to

$$x(1) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

and minimizes the control effort

$$\int_0^1 u^2(t)\ dt \quad ?$$

If so, find it.

Problem 3

Consider the problem of finding a time function $x(t)$ such that $x(0) = 1$ and $x(2\pi) = 0$, such that the functional

$$\eta = \int_0^{2\pi} (\dot{x}^2 - 2\cos(t)\,\dot{x})\ dt$$

is minimized. Use optimal control theory to solve this problem. Determine the minimum value of η.