Homework 1 (partial; more will be assigned on February 1)

1. The robot model (4 *) implies

$$\dot{x}_1 \sin(x_3) - \dot{x}_2 \cos(x_3) = 0$$

Does this mean that the system is nonholonomic ?

2. The set of matrices of the form

$$g = \begin{bmatrix} \cos(x_3) & -\sin(x_3) & x_1 \\ \sin(x_3) & \cos(x_3) & x_2 \\ 0 & 0 & 1 \end{bmatrix}$$

is a group in the sense that it ~~is~~ closed under multiplication, each element of the set has an inverse in the set, and there is a unique identity element. Verify this.

3. We denote the set in problem 2 as SE(2) (for special Euclidean matrix Lie group of rigid motions in the plane). Show that the robot model is equivalent to

$$\dot{g} = g\,(u X_1 + v X_2)$$

for suitable 3×3 matrices X_1 and X_2.
Determine X_1 and X_2. $\blacksquare$