Problem Set 1 part b < both parts to be returned Feb 8 >

1. Prove (Fredholm Alternative)

Let V and W be finite dimensional vector spaces with inner products $\langle \cdot, \cdot \rangle_V$ and $\langle \cdot, \cdot \rangle_W$ respectively. For $A : V \to W$ a linear map, define the adjoint linear map, $A^* : W \to V$ by requiring

$$\langle Av, w \rangle_W = \langle v, A^* w \rangle_V$$

for all $v \in V$ and $w \in W$. Then

$Ax = b$ has a solution

if and only if

for every $p \in \text{Kernel}(A^*)$

$$\langle p, b \rangle_W = 0$$

From this show $\text{range}(A) = \text{range}(AA^*)$

2. On page 10 of Lecture Notes 1 compute the efficiency, and determine how it changes if a series inductor of value L is added.