

Monday
March 7

Thursday
March 1
2012

ENEE 664 Spring 2012 ⁶ Homework #5 due

1. Consider $f: \mathbb{R}^2 \rightarrow \mathbb{R}^1$ defined by

$$f(x_1, x_2) = \begin{cases} \frac{x_1 x_2^2}{x_1^2 + x_2^4} & x_1 \neq 0 \\ 0 & x_1 = 0 \end{cases}$$

Show that f is Gateaux differentiable but not continuous at $x_1 = x_2 = 0$.

2. Consider $f: \mathbb{R}^2 \rightarrow \mathbb{R}^1$ defined by

$$f(x_1, x_2) = \begin{cases} 0 & \text{if } x_1 = 0 \\ \frac{2x_2 e^{-\frac{1}{x_1^2}}}{x_1^2 + \left(e^{-\frac{1}{x_1^2}}\right)^2} & \text{if } x_1 \neq 0 \end{cases}$$

Show that f is Gateaux differentiable but not Fréchet differentiable at $(0, 0)$.

3. Define $f: \mathbb{R}^2 \rightarrow \mathbb{R}^1$ by

$$f(x, y) = \operatorname{sgn}(y) \min(|x|, |y|).$$

For any $h \in \mathbb{R}^2$, show that

$$\lim_{t \rightarrow 0} \frac{1}{t} (f(th) - f(0)) = f(h).$$

Is f Fréchet differentiable?

4. Let $X = C[0, 1]$ with $\|x\| = \max_{0 \leq t \leq 1} |x(t)|$

Let $f: X \rightarrow \mathbb{R}$ be defined as

$$f(x) = \left(x\left(\frac{1}{2}\right)\right)^2$$

Find the Fréchet differential of f .

5. Consider the mapping defined on symmetric matrices by

$$K \mapsto f(K) = A^T K + K A - K B B^T K + L$$

where $A, B, L = L^T$ are given. Compute the Fréchet derivative of f at K_0 , denoted by $Df(K_0)$.

When is it invertible?