ENEE 664 Optimal Control Homework 6
due March 15 to Semih Kara (GTA).

1. On page 4 of Lecture Notes 5(a), in the EXERCISE, we assume continuity of the first partial derivatives of g, $\frac{\partial g}{\partial x}(x,t)$.

Do we need the stroger notion of uniform continuity [read up on this concept]?

2. In Theorem 2, Lecture Notes 5(b), page 7, we show that,

if $\dim(X) = m < \infty$

and for $n \leq m$, if $f, f_1, f_2, \cdots, f_n$ are linear functionals on X and if ~~for each $x \in X$~~, $\bigcap_{i=1}^{n} \operatorname{Ker}(f_i) \subset \operatorname{Ker}(f)$, then $f = \sum_{i=1}^{n} \alpha_i f_i$ for some α_i, $i = 1, 2, \dots n$.

Remove the assumption that X is finite dimensional and prove the same result.

Hint: [Consider $\tilde{f} = \sum_{i=1}^{n} f(x_i) f_i$. show that $\tilde{f} = f$ on the subspace $V = \operatorname{span}\{x_1, x_2, \cdots, x_n\}$; where $\{x_i : i = 1, 2, \dots, n\}$ satisfies $f_i(x_j) = \delta_j^i$; the Kronecker symbol. Extend this $\tilde{f}$ to all of X so that $\tilde{f} = f$ on all of X]

3. Let $X = \{ x(\cdot) : [0,1] \to \mathbb{R}^2 \mid x(t)$ is continuously differentiable in $t \}$

Let $g : X \to \mathbb{R}$ be the nonlinear functional

$$g(x) = \int_0^1 \left(x_1(t)\, \dot{x}_2(t) - x_2(t)\, \dot{x}_1(t) \right) dt$$

where $x_1(\cdot)$ and $x_2(\cdot)$ are respectively the first and second components of $x(\cdot)$.

Let $\Omega = \{ x(\cdot) \in X \mid g(x) = 0;\ x_i(0) = x_i(1) = 0,\ i = 1, 2 \}$

For a suitable norm on X, determine conditions for x to be a <u>regular point</u> of Ω

SHOW NUMERICAL ITERATIONS

4. Given $A, B > 0$ and $a, b \in \mathbb{R}$ and $A \neq B$, find (the) point in $\mathbb{R}^2$ that is closest to the origin $(0,0)$ and also lies on the ellipse

$$(x-a)^2/A + (y-b)^2/B = 1.$$

Create code for MATLAB problem

5. Consider $f(K) = K^2 - L$ defined on $n \times n$ symmetric matrices K and $L = L^T$ given. It is of interest to solve $f(K) = 0$. If L is positive semidefinite, extend the classical <u>Newton algorithm</u> to the setting when $n = 2$ and $L = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$ and find K.