Homework Set 7 ENEE 664 Spring 2014

Due date: 04/09/2014

1. Consider the following two optimal control problems

(a)
$$\min_{\substack{u_i(\cdot)\\ i=1,2}} \int_0^1 \left(u_1^2(t) + u_2^2(t)\right) dt$$

subject to
$$\dot{x}_i(t) = u_i(t) \qquad i=1,2$$
$$\dot{x}_3(t) = x_1(t)\,u_2(t) - x_2(t)\,u_1(t)$$
$$x_i(0) = x_i(1) = 0 \qquad i=1,2$$
$$x_3(0) = 0\ ; \quad x_3(1) = 1$$

(b)
$$\min_{\substack{u_i(\cdot)\\ i=1,2}} \int_0^1 \left(u_1^2(t) + u_2^2(t)\right) dt$$

subject to
$$\dot{x}_i(t) = u_i(t) \qquad i=1,2$$
$$\dot{x}_3(t) = x_1^2(t)\,u_2(t) - x_2^2(t)\,u_1(t)$$
$$x_i(0) = x_i(1) = 0 \qquad i=1,2$$
$$x_3(0) = 0\ ; \quad x_3(0) = 1$$

Investigate and compare the two problems at the level of first order necessary conditions. Is there a sense in which the two problems can be categorized in terms of levels of difficulty of finding candidate solutions?

2. Consider the problem

$$\text{Min} \int_0^1 \left(\frac{1}{2} (\dot{x}(t))^2 + \frac{1}{2} (x(t))^2 - t\, x(t) \right) dt$$

subject to $\quad x(0) = 0 \;;\; x(1) = 1$

Does this problem have a unique extremal?
If so, is it a minimum of J?

3. Show that a curve of minimum length joining two points on a sphere

$$S^2 = \{ (x, y, z) : x^2 + y^2 + z^2 = 1 \}$$

is on a great circle on the sphere.

4. Consider the problem of minimizing

$$J[x] = \int_{t_1}^{t_2} L(t, x(t), \dot{x}(t))\, dt$$

subject to $\quad x(t_1) = x_1$

Derive first order necessary conditions for this problem.

5. Show that

$$\int_0^{\pi/2} (\dot{x}(t))^2 \, dt \geq \int_0^{\pi/2} (x(t))^2 \, dt$$

for $x(0) = 0$,

using (a) optimal control theory
(b) Calculus of variations.