$$\dot{\phi} = -\gamma \; w(t)\, w^T(t)\; \phi$$

$$= -A(t)\,\phi(t)$$

$$A(t) = \gamma\; w(t)\, w(t)^T = A^T(t) \geq 0 .$$

$w(t)$ is persistently exciting if

$\exists\; \alpha_1, \alpha_2, \delta > 0$ s.t.

$$\alpha_2 \mathbb{1} \geq \int_{t_0}^{t_0+\delta} w(\tau)\, w^T(\tau)\, d\tau \geq \alpha_1 \mathbb{1}$$

$\forall\; t_0 \geq 0.$

Theorem: Suppose w is persistently exciting for $\dot{w}$ piecewise continuous. Then $\dot{\phi} = -\gamma\, w w^T \phi$ is globally exponentially stable. $\boxtimes$