2017.09.

ENEE 765 Lecture 3

When we seek to understand data via linear static models, the following representation holds:

$$y = X\beta + \varepsilon$$

Here y is a $n \times 1$ data vector, ε is a $n \times 1$ noise vector, β is a vector of parameters to be extracted from data, and each row of the $n \times k$ matrix X is an instantiation of a regressor vector. Regressor vectors may arise from data outcomes of experiments or may be assumed models, e.g.

$$x = (1, z, z^2, \ldots, z^{k-1})$$

arises when we seek to represent y as a polynomial $\sum_{i=1}^{k} \beta_i z^{i-1}$! To determine $\hat{\beta}$ an estimate of β, one might resort to least squares:

$$\hat{\beta} = \arg\min_{\beta} \sum_{i=1}^{n} \Big(y_i - \sum_{j=1}^{k} X_{ij} \beta_j\Big)^2$$

$$= \arg\min_{\beta} (y - X\beta)^T (y - X\beta)$$

This idea may be extended to the setting of <u>indirect adaptive control</u>, where a controller is adapted using a <u>running estimate</u> of a parameter in a partially known plant ~~model~~, with the aim of <u>matching</u> a known reference model.

from an identification algorithm

<u>P</u>: $\dot{y}_p = -a y_p + k_p u$ (<u>plant</u>)

<u>M</u>: $\dot{y}_m = -a y_m + r$ (<u>ref model</u>)

$a > 0$ is known but k_p is unknown

<u>Ideal Controller</u> $u(t) = \theta^* r(t)$

with $\theta^* = \frac{1}{k_p}$ if known.

<u>Adaptive Controller</u> $u(t) = \theta(t)\, r(t)$

with $\theta(t)$ <u>suitably adapted</u>

<u>E</u>: <u>example</u>: $\theta(t) = \frac{1}{\pi(t)}$ (<u>certainty equivalence</u>)

where $\pi(t)$ = running estimate of parameter k_p. <u>How do we get this?</u>

(i) Feed $u(\cdot)$ to <u>regressor system</u>

R: $\dot{w}(t) = -a\, w(t) + u(t)$

Let $\pi^* = k_p$ denote true parameter

Then

(ii) errors

$$\psi(t) = \pi(t) - \pi^* = \text{parameter error}$$

$$e_o(t) = y_p(t) - y_m(t) = \text{output error}$$

$$e_i(t) = \pi(t)\, w(t) - y_p(t)$$

$$= \text{identification error}$$

(iii) If the plant and regressor have been running for a while then the effects of plant initial condition $y(0)$ and regressor initial condition $w(0)$ die out and we can write

Because $a > 0$

$$y_p(t) \approx k_p\, w(t) = \pi^*\, w(t)$$

$$\Rightarrow \quad e_i(t) \approx (\pi(t) - \pi^*)\, w(t)$$
$$= \psi(t)\, w(t)$$

Now we obtain $\pi(t)$ by the identification algorithm

(iv) $$\dot{\pi} = \dot{\overbrace{(\pi - \pi^*)}} = \dot{\psi} = -\gamma e_i w = -\gamma \psi w^2$$

with $\gamma > 0$

Let $V = \frac{1}{2}\psi^2$ = a measure of error.

Along trajectories of the identification algorithm

$$\frac{dV}{dt} = \psi\dot{\psi}$$

$$= -\gamma\psi^2 w^2 \leq 0$$

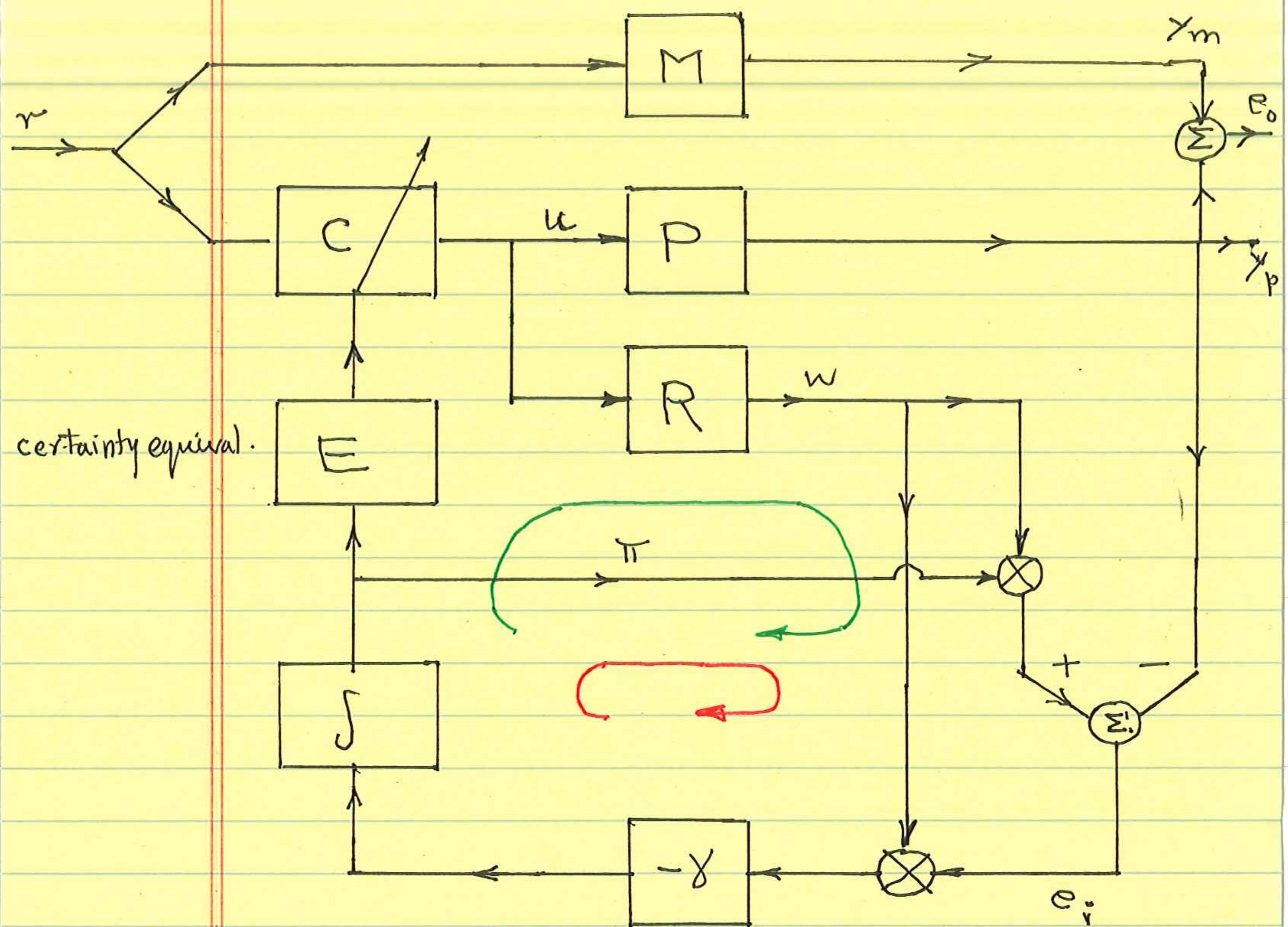
So identification algorithm has the property that it reduces the parameter estimation error. Does it reduces the error to zero?

Notice

$$\psi(t) = \psi(0)\exp\left(-\gamma\int_0^t w^2(\sigma)\,d\sigma\right)$$

If $\int_0^t w^2(\sigma)d\sigma \to \infty$ as $t \to \infty$ (condition of persistent excitation), then $\psi(t) \to 0$ as $t \to \infty$, and hence $\pi(t) \to \pi^*$ = true parameter value.

<u>BUT</u> this condition is dependent on what $r(\cdot)$ is fed to the regressor system.



Indirect Adaptive Control

Red loop : identification
Green loop : adaptive control