ENEE 765 2017 HW2 due 2017.09.28

1. Let $X \subset \mathbb{R}^n$ (closed) be a stimulus/input space and let $K : X \times X \to \mathbb{R}$ denote a kernel. Show that

(a) $K(x, x) \geq 0 \quad \forall \; x \in X$

(b) $|K(x_1, x_2)|^2 \leq K(x_1, x_1) \cdot K(x_2, x_2)$

(c) In the associated RKHS, H_K

$$|f(x)|^2 \leq K(x, x) \cdot \langle f, f \rangle_K$$

2. Consider learning a map $f : X \to \mathbb{R}$ hypothesized to lie in the RKHS, H_K associated to kernel $K : X \times X \to \mathbb{R}$. Given the data set $\{(x_i, y_i) \in X \times \mathbb{R}\}_{i=1,2,\ldots,m}$

determine the form of a minimizer of the cost

$$\boxed{C(f) := \exp\left(\frac{\lambda}{m} \sum_{i=1}^{m} (y_i - f(x_i))^2\right) + \gamma \|f\|^2_{H_K}}$$

where γ and $\lambda > 0$.

3. Consider the learning problem

$$C(f) := \frac{1}{m} \sum_{i=1}^{m} (y_i - f(x_i))^2 + \gamma \|f\|^2_{H_K}$$

suppose m grows to $(m+1)$, $(m+2)$ and so on indicating sequential arrival of data (x_i, y_i) ; $i = 1, 2, \cdots, m, (m+1), \cdots$

Suggest a method to efficiently update an estimate $f_m \in H_K$ based on

a dataset of size m to an estimate $f_{m+1} \in H_K$ recursively.

4 Read Poggio - Smale paper upto page 540, and the Simic paper closely, with a focus on Gaussian kernel.