PSK
08/29/01

ENEE 765 Adaptive Control

Lecture 1

The primary goals of this course are:

(a) To provide an exposition of the basic features of adaptive control;

(b) Formulation of convergence and performance concepts for adaptive controllers;

(c) to work out the classification of types of adaptive control and underlying principles;

(d) to exploit techniques of nonlinear analysis in understanding adaptive system behavior and to enable design of such systems.

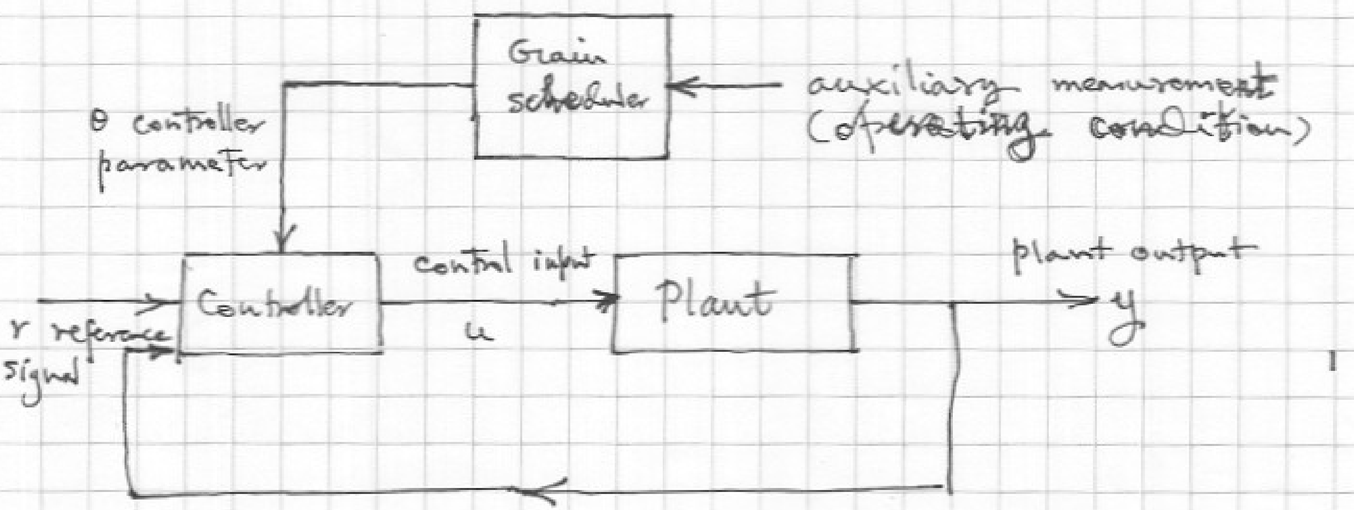
Some of the salient aspects of adaptive control include:

- Variation of gain parameters in controllers according to specific gain modification / adaptation laws
- Online identification of parameters of a system as a precursor to using parameter estimates in control laws.
- nonlinear dynamics in system + adaptive controller

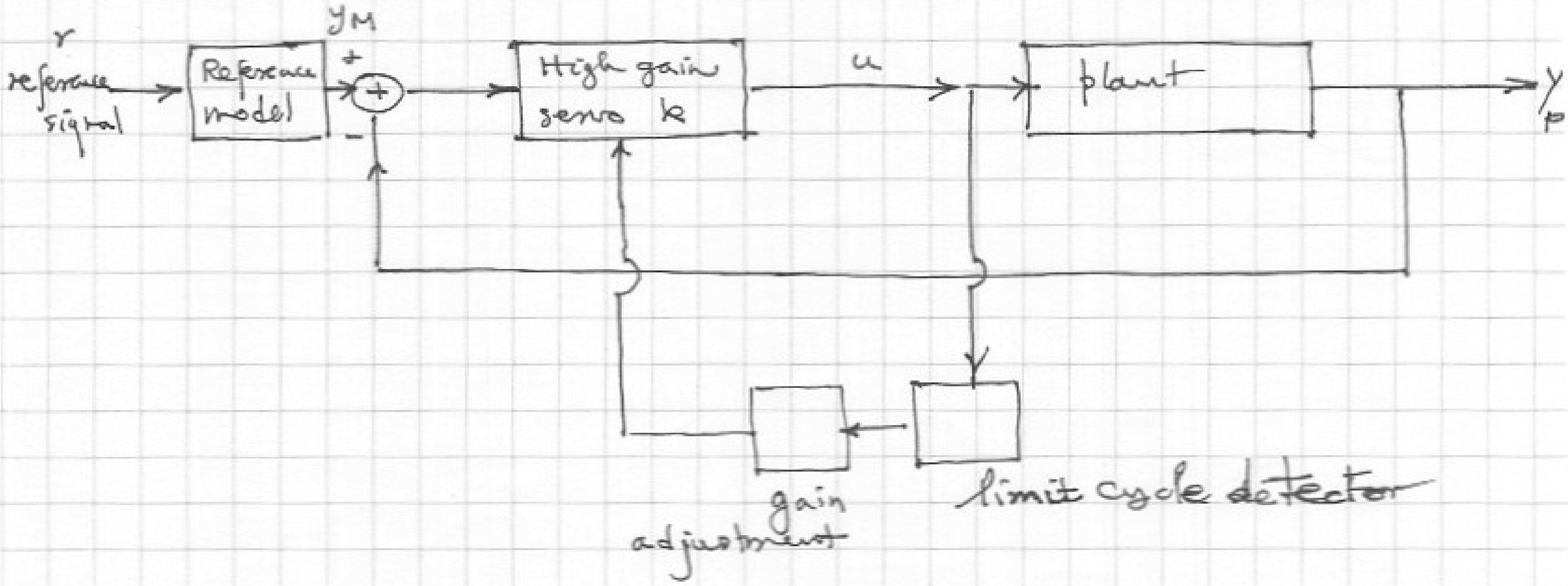
Some basic questions to ask include

1. When and why do we need adaptive control?
2. Will an adaptive controller perform at least as well as a constant gain controller?
3. Will adaptive controllers have reasonable transient behavior (or will control signals vary wildly)? (robustness)
4. Will online system identification algorithms used in adaptive control show parameter convergence? Do they need to?

A 'taxonomy' of adaptive control

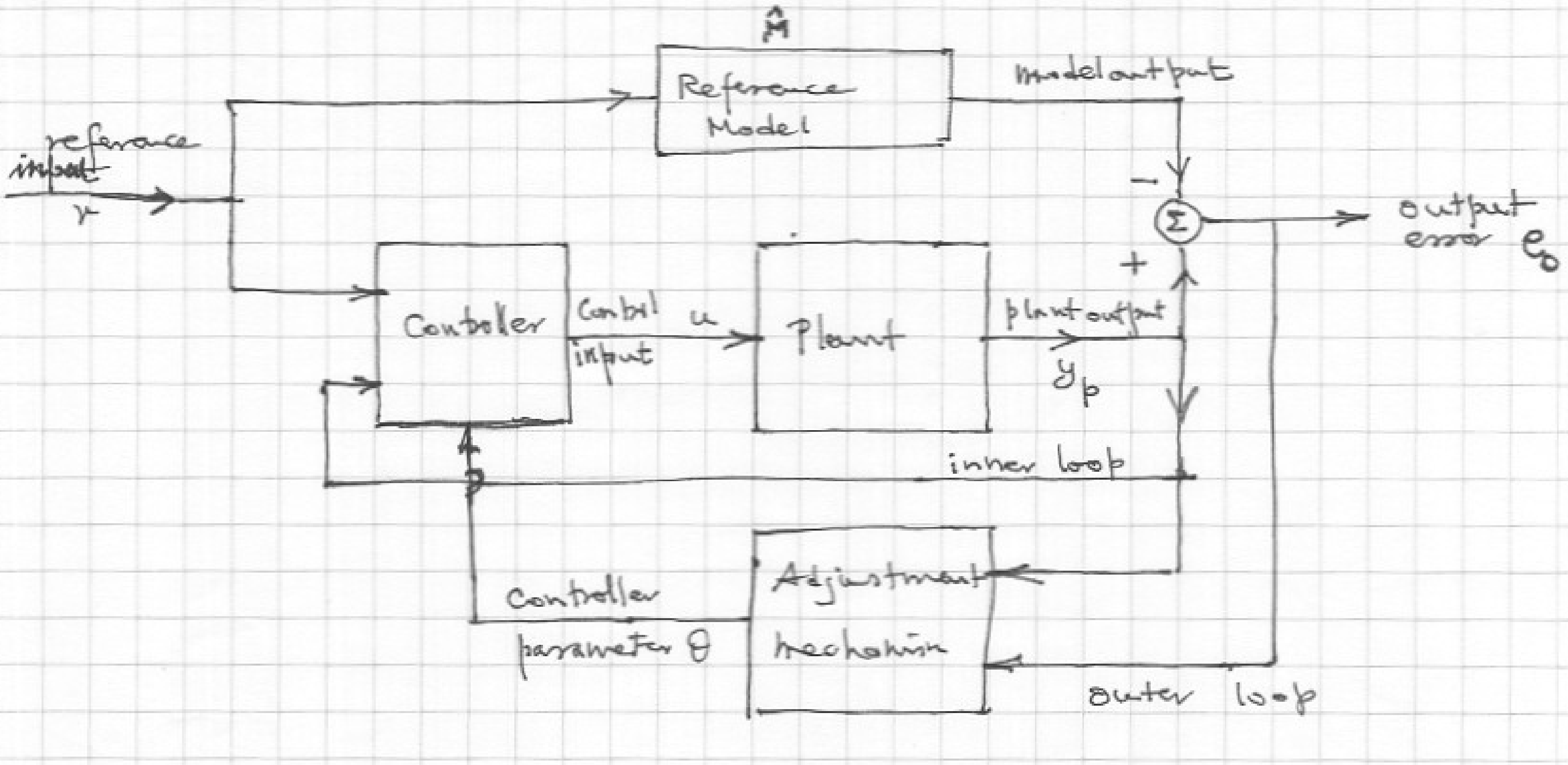


Gain scheduling Controller
(an adaptive version of diagram (ii) above)



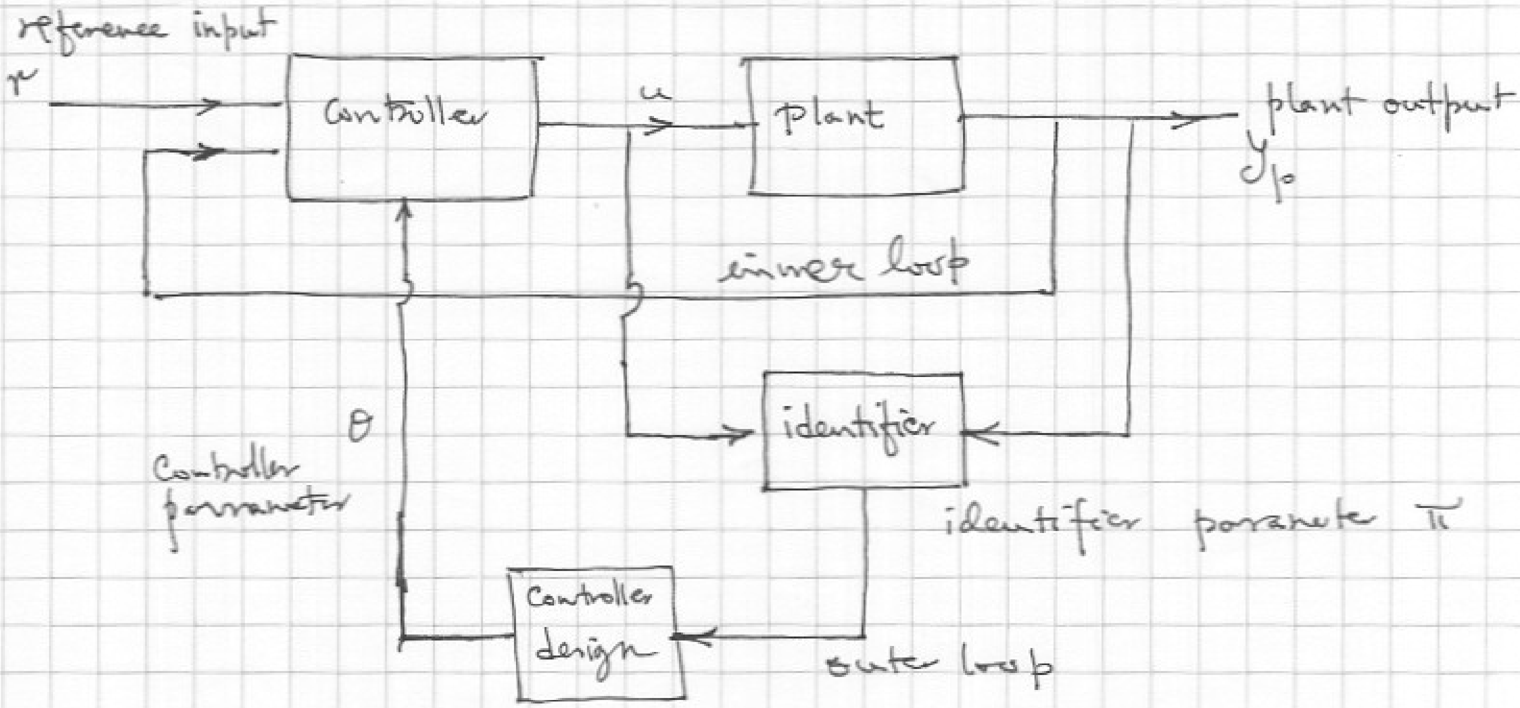
(adaptive version of diagram (iii) above)

Model Reference Adaptive System – series, high gain



(adaptive version of diagram (iv) above.

Model Reference Adaptive System – Parallel



Self tuning Controller

- Model reference adaptive schemes are aimed at <u>model matching</u>: make $y_p(t)$ look like $y_m(t)$, given $r(t)$.
- Model reference schemes are <u>direct</u>. Self tuning control schemes are <u>indirect</u> adaptive controllers.
- In MRAS no explicit system identification takes place.
- Inner loop in self-tuning control can be the same as that in MRAS.
- Gain scheduling involves <u>open-loop</u> adaptation of feedback gains.

To answer the question, "Why adaptive control?" one should re-visit some history. Early autopilots for aircraft needed adjustment for flight regime changes. This lead to gain scheduling. It became desirable to ask if a controller structure could be found that would work for a wide range of variation of parameters in the plant. This lead to the MRAS idea.

More recent concerns include asking for 'universal controllers' that assume "nothing" about the plant. These latter controllers have forms of gain adaptation rate that can be interpreted in terms of principles of extracting information regarding key plant parameters through probing / learning.

Example (Byrnes) 1

Family of systems Σ : { plant : $\dot{y} = ay + bu$, a, b unknown, $b > 0$ }

If a, b ~~when~~ were known the constant gain feedback law

$$u = -ky$$

for <u>any</u> $k > \frac{a}{b}$ stabilizes $y = 0$.

If a, b are unknown, then consider ~~the~~ update law / rule

$$\dot{k} = y^2 .$$

The closed-loop system is given by the o.d.e's

$$\dot{y} = (a - bk)y,$$
$$\dot{k} = y^2$$

(nonlinear dynamics)

How does this behave as $t \to \infty$?

Observe that

$$\phi(y, k) = y^2 + b\left(k - \frac{a}{b}\right)^2$$

is a constant along trajectories of the closed loop system:

$$\frac{d\phi}{dt} = \frac{\partial\phi}{\partial y}\dot{y} + \frac{\partial\phi}{\partial k}\dot{k}$$

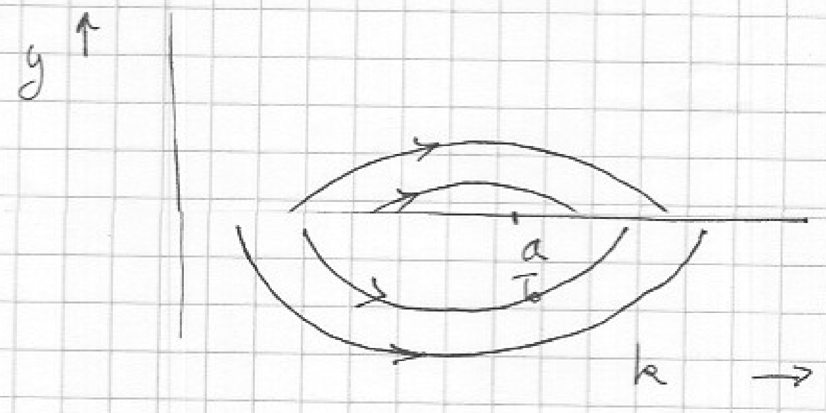
$$= 2y\dot{y} + 2b\left(k - \frac{a}{b}\right)\dot{k}$$

$$= 2y\,(ay - bky)$$

$$+ 2b\left(k - \frac{a}{b}\right)y^2$$

$$= 0.$$

The curves $\phi(y,k) =$ constant ≥ 0 are ellipses; degenerating to the single point $(y,k) = (0, \frac{a}{b})$. Every point $(0,k)$ on the k-axis is an equilibrium point of the closed loop system. The closed loop trajectories starting at (y_0, k_0) ~~always~~ (not on the k-axis), always move to the right ($\dot{k} \geq 0$) on the semi-ellipses $\phi(y,k) = \phi(y_0, k_0)$.



and it is easy to see that

$$(y(t), k(t)) \to (0, k_\infty) \quad \text{as} \quad t \to \infty$$

where $$k_\infty = \frac{a}{b} + \sqrt{\frac{\phi(y_0, k_0)}{b}}$$

So the gain does converge (to a value determined by the system parameters). The set point $y = 0$ is stabilized.

So a little phase portrait analysis tells the story! ∎

The above problem can be treated by the use of <u>Lyapunov functions</u> of slightly more general sort. Consider the function

$$V(y, k) = y^2 + b(k - \lambda)^2$$

for $\lambda > 0$.

Then, along trajectories of the closed loop system

$$\frac{dV}{dt} = 2y\dot{y} + 2b(k-\lambda)\dot{k}$$

$$= 2y(ay - bky)$$
$$+ 2b(k-\lambda)y^2$$

$$= -2y^2(\lambda b - a) \leq 0$$ provided

we pick λ such that $\lambda b > a$.

For such λ, V is a Lyapunov function, and

$$E = \left\{ (y,k) : \frac{dV}{dt}(y,k) = 0 \right\}$$

$$= \left\{ (0,k) : k \text{ arbitrary} \right\}$$

As mentioned before, every point on E is an equilibrium point of the closed loop system. In fact the largest invariant set contained in E is

$$M = E$$

See Nonlinear Controls (ENEE 661) Lecture #

By <u>LaSalle's invariance principle</u>

$$(y(t), k(t)) \longrightarrow M = E \quad \text{as } t \to \infty$$

(in fact, for arbitrary initial conditions, as