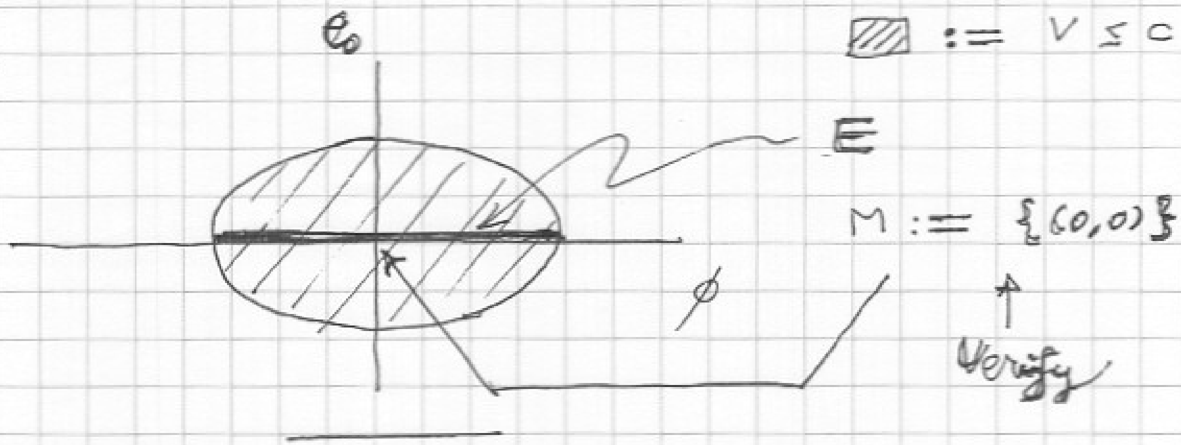

Also since

$$A = \begin{bmatrix} a_m & k_p r \\ -\gamma r & 0 \end{bmatrix}$$

$$\text{spectrum}(A) = \{ s : (s - a_m)(s - 0) + k_p \gamma r^2 = 0 \}$$

$$= \{ s : s^2 - a_m s + k_p \gamma r^2 = 0 \}$$

$$= \left\{ \frac{a_m \pm \sqrt{a_m^2 - 4 k_p \gamma r^2}}{2} \right\}$$

$\rightarrow$ real distinct roots in open l.h.p

$\hookrightarrow$ $k_p > 0$
$\gamma > 0$
$r \neq 0$

$\Rightarrow$ exponential stability

09/05/01

Model matching using indirect adaptive control (— via identification)

k_p · unknown
$a > 0$ known

$$y_m = \frac{1}{s+a} r$$

Plant output error

$$e_0 = y_p - y_m$$

$$y_p = \frac{k_p}{s+a} u$$

$$\Rightarrow \; y_p = \frac{k_p}{s+a} \theta r$$

$$u(t) = \theta(t) \, r(t)$$

$$\theta^* = \frac{1}{k_p}$$

$$w = \frac{1}{s+a} u$$

$$= \frac{1}{s+a} \theta r$$

Identifier output error

$$e_i = \pi w - y_p$$

$$\theta(t) = \frac{1}{\pi(t)} \quad \langle \text{certainty equivalence} \rangle$$

$$\pi^* = k_p \qquad y_p = \frac{k_p}{s+a}(\theta r) = k_p w = \pi^* w$$

$$\Rightarrow \; e_i = (\pi - \pi^*) w \stackrel{\Delta}{=} \psi w$$

Identifier is designed to reduce identification error $\psi = \pi - \pi^*$

$$\dot{\pi} = (\dot{\pi - \pi^*}) = \dot{\psi} = -\gamma \, e_i \cdot w = -\gamma \psi w^2$$

What is the asymptotic behavior of the

identifier?

Along trajectories of

$$\dot{\psi} = -\gamma \psi w^2$$

The function $V = \frac{1}{2}\psi^2$ satisfies

$$\frac{dV}{dt} = \psi\dot{\psi}$$
$$= -\gamma(\psi w) \leq 0 .$$

Therefore identifier update law has the property that it reduces identification error. Can we says more?

$$\psi(t) = \psi(0)\exp\left(-\gamma\int_0^t w^2(\sigma)\,d\sigma\right)$$

If $\int_0^t w^2(\sigma)\,d\sigma \to \infty$ as $t \to \infty$

then $\psi(t) \to 0$ and $\pi(t) \to \pi^*$

and $\theta(t) \longrightarrow \frac{1}{k_p}$.

Thus condition $\int_0^t w^2(\sigma)\,d\sigma \to \infty$ as $t \to \infty$ is the KEY: identifiability / persistence of excit.

How can you restate this in terms of $r(t)$ alone?

Special case (3) can be analyzed as a nonautonomous system via the methods of Lyapunov theory developed for nonautonomous systems. For a typical asymptotic stability theorem in this context, one could ask if there exist class $\mathcal{K}$ functions α_i, $i = 1, 2, 3$ such that

$$\alpha_1(\|x\|) \leq V(t, x) = \gamma e_0^2 + k_p \phi^2 \leq \alpha_2(\|x\|)$$

and

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} \cdot f \leq -\alpha_3(\|x\|).$$

Clearly, for ~~exam~~ special case (3) we cannot find a suitable class function α_3. This is a difficulty that leads into look at more advanced results in stability theory — specifically invariance theorems for nonautonomous systems.